MIXED MODEL BASED/FUZZY ADAPTIVE ROBUST CONTROLLER WITH $\mathcal{H}_\infty$ CRITERION APPLIED TO FREE-FLOATING SPACE MANIPULATORS

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Abstract— This paper deals with the problem of robust trajectory tracking control, with a guaranteed $\mathcal{H}_\infty$ performance, for free-floating manipulator systems. A new control strategy is developed based on the robot mathematical model and a fuzzy adaptive control law. The dynamic model of the free-floating space manipulator is described through the Dynamically Equivalent Manipulator approach. The fuzzy adaptive control law is based on Takagi-Sugeno model, which is proposed to estimate the behavior of parametric uncertainties and spacecraft non-modeled dynamics. A nonlinear $\mathcal{H}_\infty$ controller is formulated in order to attenuate the effect of estimation errors and external disturbances in the joint positions to be controlled. Simulation results are presented to show the effectiveness of this new approach.

Keywords— Adaptive Control, Robust Control, T-S Fuzzy System, Free-Floating Space Manipulators.

1 Introduction

Free-floating space manipulators (SM) are systems which allows the spacecraft to move freely in response to manipulator motions in order to conserve fuel or electrical power. The difficulty of recreating space conditions on Earth and the need for accurate simulation and prediction of the behavior of such systems make its dynamic modeling important. An analytical method of modeling free-floating space robots called Virtual Manipulator(VM) was developed in (Vafa and Dubowsky, 1990). The Virtual Manipulator is an inertial fixed-base robot whose first joint is a passive spherical one, representing the free-floating nature of the space manipulator. However, in this approach, only kinematic equivalence is considered. Gu and Xu proposed in (Gu and Xu, 1993) an extended robot model composed of a pseudo-arm representing the base motion resulting from six hiperthetic passive joints, and a real robot arm. Based on the VM concept, Liang et al. (Liang et al., 1996) mapped a SM to a conventional fixed-base manipulator and showed that both kinematical and dynamical properties of the space manipulator system are preserved in this mapping. This manipulator is called Dynamically Equivalent Manipulator (DEM). Adaptive control schemes were applied to a SM based on the extended robot model and on the DEM concepts in (Gu and Xu, 1993) and (Parlaktuna and Ozkan, 2004), respectively. Both schemes require the measurement of the orientation, velocity and acceleration of the free-floating base as well as a complete mathematical description of the whole system.

Two approaches to control uncertain system subject to external disturbances are usually used in controller designs: adaptive control and robust control. Adaptive controllers with parameters adaptation laws, estimate and compensate the non-modeled dynamics of the system, (Craig, 1988). On the other hand, the $\mathcal{H}_\infty$ control approach has been widely used to guarantee robustness when the system is subject to disturbances, (Chen et al., 1994).

The adaptive procedure is generally based on the linear parametrization property and also demands a precise knowledge of the model structure, considering constant or slowly-varying unknown parameters. However, non-modeled dynamics are usually present and their effects also decrease the performance of this procedure. Hence, incorporating an intelligent adaptive system to the robust controller, one obtains in a unified approach the advantages of both strategies. Fuzzy logic systems and neural network systems have been successfully applied to universally approximate mathematical models of dynamic systems, see, for instance, (Begovich et al., 2002; Cao et al., 2000; Chang, 2005; Lian et al., 2002; Purwar et al., 2005; Shaoceng et al., 2000; Takagi and Sugeno, 1985; Tseng and Chen, 2000).

In (Taveira et al., 2006), the authors presented a comparative study of three techniques of nonlinear adaptive $\mathcal{H}_\infty$ controllers applied to free-floating space manipulators. In addition to (Taveira et al., 2006), this work aims to apply a T-S fuzzy scheme to approximate only parametric uncertainties and non-modeled dynamics, such that the adaptive technique works as a complement of the nominal model. The nominal model structure of the manipulator is described through the DEM approach. Different from (Gu and

Xu, 1993) and (Parlaktuna and Ozkan, 2004), position, velocity and acceleration of the free-floating base are not included in the control law, since these variables are generally difficult to obtain.

This paper is organized as follows: the DEM concept and problem formulation are presented in Section 2; the solution for the nonlinear $\mathcal{H}_\infty$ control problem based on the robot mathematical and T-S fuzzy models is presented in Section 3; and, finally, simulation results for a two-link free-floating space manipulator are presented in Section 4.

2 Model Description and Problem Formulation

2.1 Free-Floating Space Manipulator Mapped by a Dynamically Equivalent Fixed-Base Manipulator

Consider an n-link serial-chain rigid manipulator mounted on a free-floating base and that no external forces and torques are applied on this system. Consider also the Dynamically Equivalent Manipulator (DEM) approach proposed in (Liang et al., 1996). The DEM is an $n+1$-link fixed-base manipulator with its first joint being a passive spherical one and, whose model is both kinematically and dynamically equivalent to the SM dynamics. The DEM concept not only allows us to model a free-floating space manipulator system with simple and well-understood methods but also provides data to build a real physical system. Then, a conventional manipulator system can be used to experimentally study the dynamic performance and task execution of a space manipulator system, without having to resort to complicated experimental set-ups to simulate the space environment.

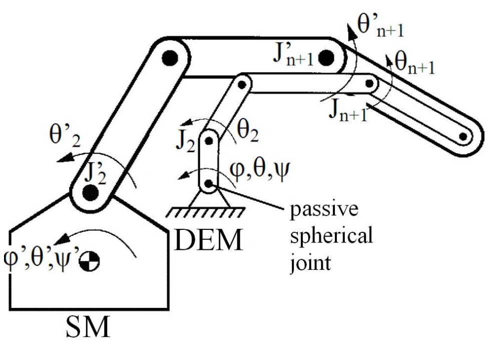

Figure 1: The space manipulator and its corresponding DEM.

Figure 1 shows the representation and the parameter notation for both SM and DEM manipulators. Let the SM parameters be identified by $'$, the links of the manipulators are numbered from 2 to $n+1$; the Z-Y-Z euler angles (ϕ, θ, ψ) represent the orientation of the SM's base and the angles of the DEM's first passive joint; J_i is the joint connecting the $(i-1)$-*th* link and i-*th* link; θ_i is the rotation of the i-*th* link around joint J_i; C_i is the center of mass of the $i-$th link; L_i is the vector connecting ${J_i}'$ and ${C_i}'$; R_i is the vector connecting ${C_i}'$ and ${J_{i+1}}'$; l_{ci} is the vector connecting J_i and C_i; and W_i is the vector connecting J_i and J_{i+1}.

Locating the passive spherical joint at the center of mass of the SM and considering that the manipulator operates in the same environment as does the SM, i.e., in the absence of gravity, the kinematic and dynamical parameters of the DEM can be found from the SM parameters as (Liang et al., 1996)

$$
\begin{aligned}
m_i &= \frac{M_t^2 {m_i}'}{\sum\limits_{j=1}^{i-1} {m_j}' \sum\limits_{j=1}^{i} {m_j}'}, && i = 2, ..., n+1, \\
I_i &= {I_i}', && i = 1, ..., n+1, \\
W_1 &= R_1 \frac{\sum\limits_{j=1}^{i} {m_j}'}{M_t}, && \\
W_i &= R_i \frac{\sum\limits_{j=1}^{i} {m_j}'}{M_t} + L_i \frac{\sum\limits_{j=1}^{i-1} {m_j}'}{M_t}, && i = 2, ..., n+1, \\
l_{c1} &= 0, && \\
l_{ci} &= L_i \frac{\sum\limits_{j=1}^{i-1} {m_j}'}{M_t}, && i = 2, ..., n+1,
\end{aligned}
\tag{1}
$$

where M_t is the total mass of the SM. Observe that the mass of the passive joint, m_1, is not defined by the equivalence properties.

Since the gravitational forces do not act upon the DEM system and that no flexible components are considered, from Lagrange theory, the dynamic equations of the DEM are given by

$$M(q)\ddot{q} + h(q, \dot{q}) = \tau, \tag{2}$$

where $q = [\ \phi \ \ \theta \ \ \psi \ \ \theta_2 \ \ \cdots \ \ \theta_{n+1}\ ]^T$ are the generalized coordinates, $M(q) \in \mathbb{R}^{n+3 \times n+3}$ is the symmetric positive definite inertia matrix, $h(q, \dot{q}) \in \mathbb{R}^{n+3}$ is the vector of the Coriolis and centrifugal forces, and $\tau = [\ 0 \ \ 0 \ \ 0 \ \ \tau_2 \ \ \cdots \ \ \tau_{n+1}\ ]^T$ is the torque vector acting upon the joints of the DEM.

2.2 Problem Formulation

As we are dealing with a free-floating space manipulator, it is considered that only the active joints of the DEM are controlled, with the passive spherical joint not locked. In this case, the passive joint dynamics intervenes with the control of the manipulator active joints.

Let q be partitioned as $q = [\ q_b^T \ \ q_m^T\ ]^T$, where the indexes b and m represent the passive spherical joint (base) and the active joints (manipulator), respectively. Let $h(q, \dot{q})$ be written as $h(q, \dot{q}) = [\ h_b^T \ \ h_m^T\ ]^T = C(q, \dot{q})\dot{q}$. Define $\delta = [\ \delta_b^T \ \ \delta_m^T\ ]^T$ as a vector representing the sum of parametric uncertainties of the system and a finite energy exogenous disturbance. (2) can be rewritten as

$$\begin{bmatrix} 0 \\ \tau_m \end{bmatrix} + \begin{bmatrix} \delta_b \\ \delta_m \end{bmatrix} = \begin{bmatrix} M_{bb} & M_{bm} \\ M_{mb} & M_{mm} \end{bmatrix} \begin{bmatrix} \ddot{q}_b \\ \ddot{q}_m \end{bmatrix} + \begin{bmatrix} C_{bb} & C_{bm} \\ C_{mb} & C_{mm} \end{bmatrix} \begin{bmatrix} \dot{q}_b \\ \dot{q}_m \end{bmatrix}, \quad (3)$$

where $M_{bb}(q_m) \in \mathbb{R}^{3\times3}$, $M_{bm}(q_m) \in \mathbb{R}^{3\times n}$, $M_{mb}(q_m) \in \mathbb{R}^{n\times3}$, $M_{mm}(q_m) \in \mathbb{R}^{n\times n}$, $C_{bb}(q_m, \dot{q}) \in \mathbb{R}^{3\times3}$, $C_{bm}(q_m, \dot{q}) \in \mathbb{R}^{3\times n}$, $C_{mb}(q_m, \dot{q}) \in \mathbb{R}^{n\times3}$, $C_{mm}(q_m, \dot{q_m}) \in \mathbb{R}^{n\times n}$ and $\tau_m \in \mathbb{R}^n$.

The controller is designed for the SM considering the DEM model.

Let q_m^d and $\dot{q}_m^d \in \mathbb{R}^n$ be the desired reference trajectory and the corresponding velocity for the controlled joints, respectively. The state tracking error is defined as

$$\tilde{x}_m = \begin{bmatrix} \dot{q}_m - \dot{q}_m^d \\ q_m - q_m^d \end{bmatrix} = \begin{bmatrix} \dot{\tilde{q}}_m \\ \tilde{q}_m \end{bmatrix}. \quad (4)$$

The variables q_m^d, $\dot{q}_m^d$, and $\ddot{q}_m^d$, the desired acceleration, are assumed to be within the physical and kinematics limits of the control system and there exists no reference trajectory for the base.

Consider the following state transformation, (Chen et al., 1994)

$$\tilde{z} = T_0\tilde{x}_m = \begin{bmatrix} T_{11} & T_{12} \\ 0 & I \end{bmatrix} \begin{bmatrix} \dot{\tilde{q}}_m \\ \tilde{q}_m \end{bmatrix}, \quad (5)$$

where T_{11}, $T_{12} \in \mathbb{R}^{n\times n}$ are constant matrices to be determined. From (3), (4) and (5), state space representation of the DEM is given by

$$\dot{\tilde{x}}_m = A_T(q_m, \dot{q}_m)\tilde{x}_m + B_T(q_m)u + B_T(q_m)w, \quad (6)$$

where

$$A_T(q_m, \dot{q}_m) = T_0^{-1} \begin{bmatrix} -M_{mm}^{-1}C_{mm} & 0 \\ T_{11}^{-1} & -T_{11}^{-1}T_{12} \end{bmatrix} T_0,$$
$$B_T(q_m) = T_0^{-1} \begin{bmatrix} M_{mm}^{-1} \\ 0 \end{bmatrix},$$
$$w = T_{11}\delta_m,$$
$$u = T_{11}(\tau_m - F(x_{e_m}) - M_{mb}\ddot{q}_b - C_{mb}\dot{q}_b),$$

and

$$F(x_{e_m}) = M_{mm}(\ddot{q}_m^d - T_{11}^{-1}T_{12}\dot{\tilde{q}}_m) + C_{mm}(\dot{q}_m^d - T_{11}^{-1}T_{12}\tilde{q}_m),$$

with

$$x_{e_m} = \begin{bmatrix} \ddot{q}_m^d & \dot{q}_m^d & q_m^d & \dot{q}_m & q_m \end{bmatrix}.$$

The applied torques can be computed by

$$\tau_m = T_{11}^{-1}\bar{u} + F(x_{e_m}) + M_{mb}\ddot{q}_b + C_{mb}\dot{q}_b, \quad (7)$$

where $\bar{u}$ is the control law to be provided by the nonlinear $\mathcal{H}_\infty$ controller.

In fact, parametric uncertainties are also present in the known nominal value of the term $F(x_{e_m})$. The actual value is then written as $F_r(x_{e_m}) = F(x_{e_m}) + \Delta F(x_{e_m})$. In this case, (7) can be rewritten as

$$\tau_m = T_{11}^{-1}\bar{u} + F(x_{e_m}) + \Delta F(x_{e_m}) + M_{mb}\ddot{q}_b + C_{mb}\dot{q}_b. \quad (8)$$

In this approach, the term

$$E(x_e) = \Delta F(x_{e_m}) + M_{mb}\ddot{q}_b + C_{mb}\dot{q}_b,$$

includes the parametric uncertainties and the free-floating base dynamics. It is considered unknown and it is estimated by a T-S fuzzy model. It is important to note that within this strategy, measured values for velocity and acceleration of the free-floating base are not necessary.

3 Fuzzy Adaptive Robust Controller Design

The nonlinear $\mathcal{H}_\infty$ control procedure proposed in this paper is developed based on the combination of two approaches: one based on the nominal model and other based on fuzzy system via the Takagi-Sugeno methodology.

3.1 Takagi-Sugeno Fuzzy Model

In general, a fuzzy system consists of four parts: the fuzzifier, the fuzzy rule base, fuzzy inference engine and the defuzzifier. The fuzzifier is a mapping from the input universe of discourse $U \subset \mathbb{R}^r$ to the fuzzy sets defined on U. There exist two factors which determine a fuzzification interface: (i) the number of fuzzy sets defined on the input universe of discourse and (ii) the membership functions related to these fuzzy sets. The fuzzy rule base is a set of linguistic statements in the form of

IF *premises are satisfied,*
THEN *consequences are inferred.*

The fuzzy inference engine is the decision making logic which employs fuzzy rules from the fuzzy rule base to determine the corresponding output to the fuzzified inputs.

The T-S fuzzy model is caracterized by a fuzzy rule base with functional consequences instead of fuzzy consequences, as

IF u_1 *is* A_{11} ***and*** u_2 *is* A_{12} ... ***and*** u_r *is* A_{1r},
THEN $y_1 = \lambda_{10} + \lambda_{11}u_1 + \lambda_{12}u_2 + \ldots + \lambda_{1r}u_r$.
$\vdots$
IF u_1 *is* A_{k1} ***and*** u_2 *is* A_{k2} ... ***and*** u_r *is* A_{kr},
THEN $y_k = \lambda_{k0} + \lambda_{k1}u_1 + \lambda_{k2}u_2 + \ldots + \lambda_{kr}u_r$.

where A_{ij}, $j = 1, \ldots, r$ and $i = 1, \ldots, k$, are linguistic variables refered to fuzzy sets defined on the input spaces U_1, U_2, ..., U_r; u_1, u_2, ..., u_r

are input variables values and k is the number of fuzzy rules.

The inferred output from the T-S method is *crisp* (hence, it does not need a defuzzifier) and it is defined by the weighed average of outputs y_i from each linear subsystem implied

$$
\begin{aligned}
y &= \frac{\sum_{i=1}^{k} \mu_i y_i}{\sum_{j=1}^{k} \mu_j} \qquad (9)\\
&= \frac{\sum_{i=1}^{k} \mu_i(\lambda_{i0} + \lambda_{i1}u_1 + \lambda_{i2}u_2 + \ldots + \lambda_{ir}u_r)}{\sum_{j=1}^{k} \mu_j}
\end{aligned}
$$

where μ_i is the freedom degree of i-th rule, defined as the minimum among the grade of membership associated to the entries in the activated fuzzy sets by the i-th rule

$$
\mu_i := A_{i1}(u_1) \wedge A_{i2}(u_2) \wedge \ldots \wedge A_{ir}(u_r). \qquad (10)
$$

Thus, considering $\tilde{x}_m = [\tilde{q}_m \;\; \dot{\tilde{q}}_m]^T$ the fuzzy inputs and $A(\tilde{x}_m) := [A_1(\tilde{q}_m) \;\; A_2(\dot{\tilde{q}}_m)]$ composed of fuzzy sets defined for the fuzzified inputs, a fuzzy system for functional estimation of the term $E(x_e)$ based on T-S method is defined as

$$
\begin{aligned}
\hat{E}(\tilde{x}_m, A(\tilde{x}_m), \Lambda) :&= \begin{bmatrix} \hat{E}_1(\tilde{x}_m^1, A(\tilde{x}_m), \Lambda_1) \\ \vdots \\ \hat{E}_n(\tilde{x}_m^n, A(\tilde{x}_m), \Lambda_n) \end{bmatrix} \\
&= \begin{bmatrix} \xi_1 \Lambda_1 \\ \vdots \\ \xi_n \Lambda_n \end{bmatrix} \\
&= \begin{bmatrix} \xi_1 & 0 & \ldots & 0 \\ 0 & \xi_2 & \vdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \xi_n \end{bmatrix} \begin{bmatrix} \Lambda_1 \\ \Lambda_2 \\ \vdots \\ \Lambda_n \end{bmatrix} \\
&= \Xi\Lambda, \qquad (11)
\end{aligned}
$$

with

$$
\xi_i = \frac{1}{\sum_{j=1}^{k} \mu_j^i} \begin{bmatrix} \mu_1^i & \mu_1^i \tilde{x}_m^{i^T} \\ \vdots & \vdots \\ \mu_k^i & \mu_k^i \tilde{x}_m^{i^T} \end{bmatrix}
$$

and

$$
\Lambda_i = \begin{bmatrix} \lambda_{10}^i & \ldots & \lambda_{1r}^i & \cdots & \lambda_{k0}^i & \ldots & \lambda_{kr}^i \end{bmatrix}^T .
$$

3.2 Nonlinear $\mathcal{H}_\infty$ Control

Given a desired disturbance attenuation level γ, the problem of the mixed model based/fuzzy adaptive robust $\mathcal{H}_\infty$ control proposed in this paper, has a solution if there exist a state feedback dynamic controller

$$
\begin{aligned}
\dot{\Lambda} &= \alpha(t, \tilde{x}_m), \qquad (12)\\
\tau_m &= \tau_m(t, \Lambda, \tilde{x}_m), \qquad (13)
\end{aligned}
$$

such that the following performance index is achieved, for any initial condition

$$
\begin{aligned}
\int_0^T \left(\tilde{x}_m^T Q \tilde{x}_m + \bar{u}^T R \bar{u}\right) dt \leq \; & \tilde{x}_m^T(0) P_0 \tilde{x}_m(0) \\
& + \tilde{\Lambda}^T(0) Z_0 \tilde{\Lambda}(0) + \gamma^2 \int_0^T (w^T w) dt, \qquad (14)
\end{aligned}
$$

where $Q = Q^T > 0, R = R^T > 0, P_0 = P_0^T > 0$, and $Z_0 = Z_0^T > 0$, while $\tilde{\Lambda} = \Lambda - \Lambda^*$ is the estimation error from the fuzzy system, where * denotes the optimum of Λ.

Considering the nonlinear $\mathcal{H}_\infty$ control via game theory, (Chen et al., 1994), let

$$
\bar{u} = -R^{-1} B^T T_0 \tilde{x} \qquad (15)
$$

be the optimal control input, with $B = [I \mid 0]^T$ and T_0 being the solution of the following algebraic equation

$$
\begin{bmatrix} 0 & K \\ K & 0 \end{bmatrix} - T_0^T B \left(R^{-1} - \frac{1}{\gamma^2} I\right)^{-1} B^T T_0 + Q = 0, \qquad (16)
$$

such that $K > 0$, $R < \gamma^2 I$ and

$$
T_0 = \begin{bmatrix} T_{11} & T_{12} \\ 0 & I \end{bmatrix} = \begin{bmatrix} R_1^T Q_1 & R_1^T Q_2 \\ 0 & I \end{bmatrix},
$$

considering R_1 the result of the Cholesky factorization

$$
R_1^T R_1 = \left(R^{-1} - \frac{1}{\gamma^2} I\right)^{-1}
$$

and the positive definite symmetric matrix Q factorized as

$$
Q = \begin{bmatrix} Q_1^T Q_1 & Q_{12} \\ Q_{12}^T & Q_2^T Q_2 \end{bmatrix}.
$$

The matrices Q and R are defined by the designer thus the restrictions are preserved.

Hence, the control law

$$
\begin{aligned}
\dot{\Lambda} &= -Z^{-T} \Xi^T T_{11} B^T T_0 \tilde{x}_m, \qquad (17)\\
\tau_m &= T_{11}^{-1} \bar{u} + F(x_{e_m}) + \Xi\Lambda, \qquad (18)
\end{aligned}
$$

is the solution for the problem of the mixed model based/fuzzy adaptive robust $\mathcal{H}_\infty$ control.

Remark 3.1 *With the solution given by the adaptive control law (17), that depends on the state tracking error $\tilde{x}_m$, the fuzzy system estimates the parametric uncertainties and the non-modeled dynamics (which includes the base dynamic of the space robot).*

4 Results

For validation purpose, the mixed model based/fuzzy adaptive robust controller with $\mathcal{H}_\infty$ performance is applied to a free-floating space planar manipulator with two links, whose nominal parameters are given in Table 1. The corresponding DEM is a fixed-base, three-link, planar manipulator whose first joint is configured as passive, $q_m = [q_2 \quad q_3]^T$ are the joints to be controlled. Its structure is based on the experimental fixed-base manipulator UArmII (Underactuated Arm II), designed and built by H. Ben Brown, Jr. of Pittsburgh, PA, USA, whose nominal parameters are given in Table 2. The nominal matrices $M(q)$ and $C(q, \dot{q})$ for the DEM can be found in (Liang et al., 1996).

Table 1: SM Parameters

Body	m_i'	I_i'	R_i	L_i
	(kg)	(kgm^2)	(m)	(m)
Base	4.816	0.008251	0.253	0
Link 2	0.618	0.0075	0.118	0.12
Link 3	0.566	0.006	0.126	0.085

Table 2: DEM Parameters

Body	m_i	I_i	W_i	lc_i
	(kg)	(kgm^2)	(m)	(m)
Link 1	1.932	0.008251	0.203	0
Link 2	0.850	0.0075	0.203	0.096
Link 3	0.625	0.006	0.203	0.077

A trajectory tracking task is defined for the space manipulator joints, characterized by initial conditions $q_m(0) = [20° \quad -40°]$ and final conditions $q_m(t_f) = [80° \quad -10°]$, with $t_f = 3s$. The reference trajectory, q_m^d, is a fifth degree polynomial. During the simulation, a limited disturbance, initializing in $t = 1s$, was introduced in the following form

$$\tau_d = \begin{bmatrix} 0.035e^{-2t}\sin(2\pi t) \\ 0.015e^{-2t}\sin(2\pi t) \end{bmatrix}.$$

Parametric uncertainties are also included in the DEM model as a multiplicative error of 0.7. A set of fuzzy systems is defined as

$$\hat{E}(\tilde{x}_m, A(\tilde{x}_m), \Lambda) := \begin{bmatrix} \hat{E}_1([\tilde{q}_m^1 \quad \dot{\tilde{q}}_m^1], A(\tilde{x}_m), \Lambda_1) \\ \hat{E}_2([\tilde{q}_m^2 \quad \dot{\tilde{q}}_m^2], A(\tilde{x}_m), \Lambda_2) \end{bmatrix}$$

where $\hat{E}_1(.)$ and $\hat{E}_2(.)$ correspond to the estimate of the uncertain part of dynamic behavior of joints 2 and 3, respectively. The fuzzy sets $A(\tilde{x}_m)$ are defined to the universe of discourse of position errors, $u_1 = \tilde{q}_m \in U_1$, and to the universe of discourse of velocity errors, $u_2 = \dot{\tilde{q}}_m \in U_2$, as shown in Figure 2.

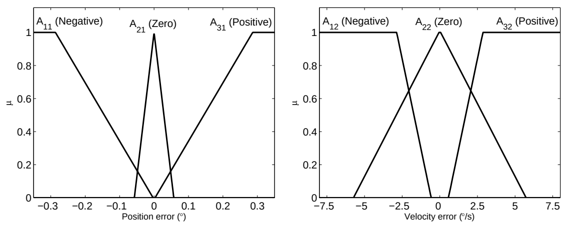

Figure 2: Fuzzy sets $A_1(\tilde{q}_m)$ and $A_2(\dot{\tilde{q}}_m)$.

The fuzzy rule base is given by

$$\begin{aligned}
R_1 &: \textbf{IF}(u_1 \text{ is } A_{11}) \text{ and } (u_2 \text{ is } A_{12}) \textbf{ THEN } y_1 \\
R_2 &: \textbf{IF}(u_1 \text{ is } A_{11}) \text{ and } (u_2 \text{ is } A_{22}) \textbf{ THEN } y_2 \\
R_3 &: \textbf{IF}(u_1 \text{ is } A_{11}) \text{ and } (u_2 \text{ is } A_{32}) \textbf{ THEN } y_3 \\
R_4 &: \textbf{IF}(u_1 \text{ is } A_{21}) \text{ and } (u_2 \text{ is } A_{12}) \textbf{ THEN } y_4 \\
R_5 &: \textbf{IF}(u_1 \text{ is } A_{21}) \text{ and } (u_2 \text{ is } A_{22}) \textbf{ THEN } y_5 \\
R_6 &: \textbf{IF}(u_1 \text{ is } A_{21}) \text{ and } (u_2 \text{ is } A_{32}) \textbf{ THEN } y_6 \\
R_7 &: \textbf{IF}(u_1 \text{ is } A_{31}) \text{ and } (u_2 \text{ is } A_{12}) \textbf{ THEN } y_7 \\
R_8 &: \textbf{IF}(u_1 \text{ is } A_{31}) \text{ and } (u_2 \text{ is } A_{22}) \textbf{ THEN } y_8 \\
R_9 &: \textbf{IF}(u_1 \text{ is } A_{31}) \text{ and } (u_2 \text{ is } A_{32}) \textbf{ THEN } y_9
\end{aligned}$$

The $\mathcal{H}_\infty$ controller is characterized by a desired attenuation level $\gamma = 2$. The selected weighting matrices are $Q_1 = 0.1225I_2$, $Q_2 = 9I_2$, $Q_{12} = 0$ and $R = 0.5I_2$. The adaptive control law to adjust y_i is implemented based on (17).

Figure 3 presents the result for joint trajectory tracking of the space manipulator, including the free-floating base movement and the position errors during the task procedure. Figure 4 presents the behavior of the joints velocities and the velocities tracking errors. Figure 5 presents the applied torques behavior. The simulation results have shown an interesting performance of the fuzzy based adaptive nonlinear $\mathcal{H}_\infty$ controller. One can observe that in presence of disturbances the position and velocity errors remain small.

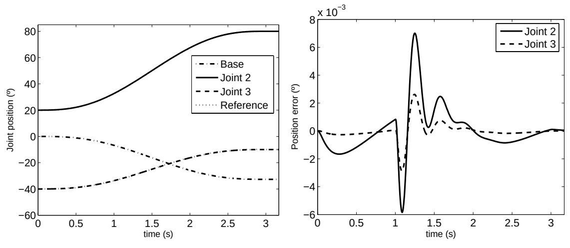

Figure 3: Joints positions and position errors.

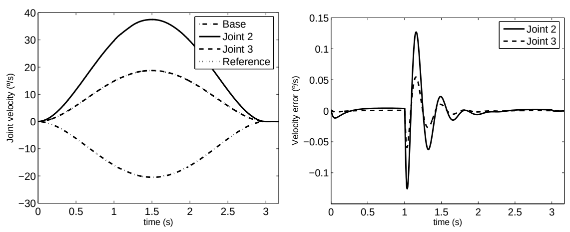

Figure 4: Joints velocities and velocity errors.

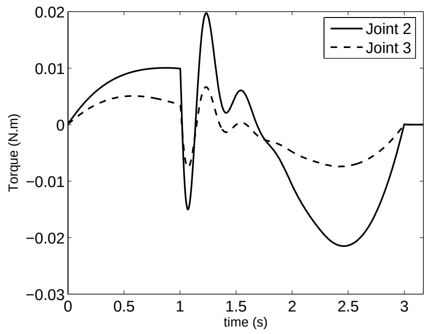

Figure 5: Applied torques.

5 Conclusion

In this paper, the problem of tracking control with a guaranteed $\mathcal{H}_\infty$ performance is solved for free-floating space manipulator systems with plant uncertainties, non-modeled dynamics and external disturbances. The new control strategy proposed considers a mixed model based/fuzzy adaptive approach. To complement the mathematical model, a fuzzy system based on Takagi-Sugeno method was applied to estimate the dynamic behavior of the free-floating base and the parametric uncertainties. Note that fuzzy approach does not demand any position, velocity or acceleration value from the free-floating base, which is a very interesting result since values as the spacecraft velocity or acceleration are not easy to obtain. The simulated results show the robustness of this new approach.

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